

### Problem-Solving Strategy 28.2 Ampere's Law

**IDENTIFY** *the relevant concepts:* Like Gauss's law, Ampere's law is most useful when the magnetic field is highly symmetric. In the form  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ , it can yield the magnitude of  $\vec{B}$  as a function of position if we are given the magnitude and direction of the field-generating electric current.

**SET UP** *the problem* using the following steps:

1. Determine the target variable(s). Usually one will be the magnitude of the  $\vec{B}$  field as a function of position.
2. Select the integration path you will use with Ampere's law. If you want to determine the magnetic field at a certain point, then the path must pass through that point. The integration path doesn't have to be any actual physical boundary. Usually it is a purely geometric curve; it may be in empty space, embedded in a solid body, or some of each. The integration path has to have enough *symmetry* to make evaluation of the integral possible. Ideally the path will be tangent to  $\vec{B}$  in regions of interest; elsewhere the path should be perpendicular to  $\vec{B}$  or should run through regions in which  $\vec{B} = 0$ .

**EXECUTE** *the solution* as follows:

1. Carry out the integral  $\oint \vec{B} \cdot d\vec{l}$  along the chosen path. If  $\vec{B}$  is tangent to all or some portion of the path and has the same magnitude  $B$  at every point, then its line integral is the product of  $B$  and the length of that portion of the path. If  $\vec{B}$  is perpendicular to some portion of the path, or if  $\vec{B} = 0$ , that portion makes no contribution to the integral.
2. In the integral  $\oint \vec{B} \cdot d\vec{l}$ ,  $\vec{B}$  is the *total* magnetic field at each point on the path; it can be caused by currents enclosed *or not enclosed* by the path. If *no* net current is enclosed by the path, the field at points on the path need not be zero, but the integral  $\oint \vec{B} \cdot d\vec{l}$  is always zero.

3. Determine the current  $I_{\text{encl}}$  enclosed by the integration path. A right-hand rule gives the sign of this current: If you curl the fingers of your right hand so that they follow the path in the direction of integration, then your right thumb points in the direction of positive current. If  $\vec{B}$  is tangent to the path everywhere and  $I_{\text{encl}}$  is positive, the direction of  $\vec{B}$  is the same as the direction of integration. If instead  $I_{\text{encl}}$  is negative, is  $\vec{B}$  in the direction opposite to that of the integration.
4. Use Ampere's law  $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$  to solve for the target variable.

**EVALUATE** *your answer*: If your result is an expression for the field magnitude as a function of position, check it by examining how the expression behaves in different limits.